# 1 Background

The `geodesy` subpackage is primarily used in ROPP to convert the vertical coordinate between geometric height ($h$) and geopotential height ($Z$).

GSR-02 discusses the implementation of the conversion routines used in ROPP-2. These are based on the Somigliana equation and its implementation as described by Mahoney (2001).

Here, we compare geodetic calculations performed in ROPP and those in an independent software package (Invert, provided by Michael Gorbunov as a VS with th GRAS SAF). Both methods use the Somigliana equation, but the Invert code is apparently more 'complete' (complex) than the ROPP implementation. The origin of the Invert expressions is not clearly documented in the code.

## 1.1 Gravity

Mahoney (2001) used Somigliana's equation to derive the normal gravity on an ellipsoid surface at a given latitude $\phi$.

$$g_s(\phi) = g_e(\frac{1 + k_s sin^2\phi}{\sqrt{1 - e^2 sin^2\phi}}) \tag{1.1}$$

where $g_e$ is the gravity at the equator (9.7803253359 ms$^{-2}$), and $k_s$ is Somigliana's constant related to the Earth shape and gravity at the equator and pole (1.931853$\times10^{-3}$) and $e$ is the eccentricity (0.081819). This is implemented in `ropp_utils/geodesy/gravity.f90`.

In the Invert code (`Lib/Earth.f90`), surface gravity is given by a second-order series expansion of the Somagliana equation (see Li and Götze 2001).

$$g_s(\phi) = g_e\left(1 + f_2 \sin^2\phi - \frac{1}{4} f_4 \sin^2 2\phi\right) \tag{1.2}$$

with $GM = 3.986004415 \times 10^{14}$ as the Earth gravity constant, $R_e = 6378.1353$ km as the equatorial semiaxis and

$$g_e = \frac{GM}{R_e^2\left(1 - f + \frac{3m}{2} - \frac{15mg}{14}\right)} \tag{1.3}$$

$$f_2 = -f + \frac{5m}{2} - \frac{17fm}{14} + \frac{15m^2}{4} = 0.0053027778 \tag{1.4}$$

$$f_4 = -\frac{f^2}{2} + \frac{5fm}{2} = 2.3297322 \times 10^{-05} \tag{1.5}$$

Li and Götze equate $f_2$ to the fractional difference between the equatorial and polar surface gravity (value = 0.00530247).

Figure 1.1 shows the difference between $g_s(\phi)$ values calculated using ROPP (Equation 1.1) and the Invert code (Equation 1.2). A constant offset of about -0.000269 ms$^{-2}$ exists between the ROPP and Invert gravity at all latitudes. Note this difference is larger than the difference between Somigliana and

Smithsonian table approaches discussed in GSR-02. This offset results from the difference between $g_e$ values assumed. i.e.

$$(\frac{1+k_s sin^2\phi}{\sqrt{1-e^2 sin^2\phi}}) \approx \left(1+f_2\sin^2\phi-\frac{1}{4}f_4\sin^2 2\phi\right) \tag{1.6}$$

If a constant value of $g_e$=9.780325339 ms$^{-2}$ is used in Equation 1.2 rather than that calculated using Equation 1.3 then there is negligible difference between the ROPP and Invert results.

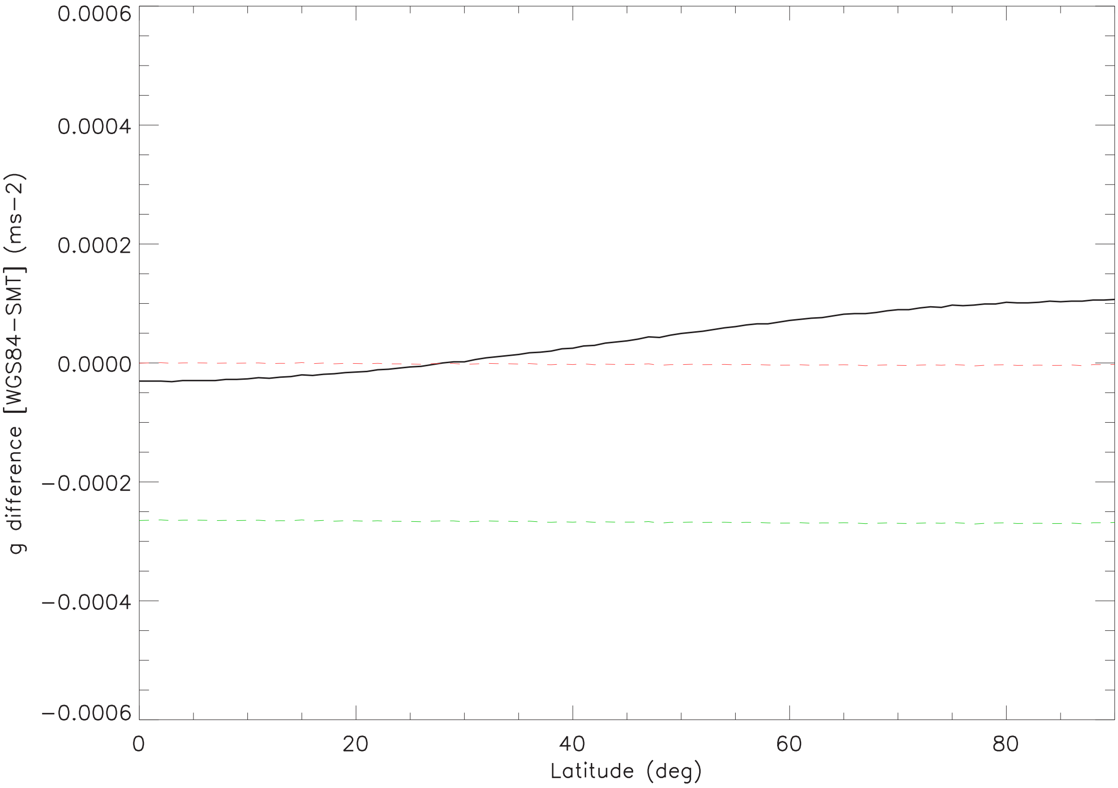



**Figure 1.1:** Black: Difference between gravity computed using Somigliana's equation (Equation 1.1) for WGS-84 ellipsoid and the values derived using the Smithsonian Meteorological Tables (see GSR-02). Green: Difference between Equation 1.1 and 1.2) as implemented in Invert. Red: Difference between Equation 1.1 and 1.2) using a value of 9.7803253359 ms$^{-2}$ for $g_e$ rather than that computed from Equation 1.3.

- What is the origin of the expressions used in the Invert code
- Why is there such a 'large' discrepancy between the calculated $g_e$ value and the established constant assumed in other applications?
- The simpler ROPP implementation of the $g_s(\phi)$ calculation apparently gives more 'accurate' results than Invert?

## 1.2 Earth radius

From Somigliana's equation, Mahoney (2001) shows that

$$R_s(\phi) = \frac{a}{1+f+m-2f\sin^2\phi} \tag{1.7}$$

where $a$ is the semi-major axis (6378.1370 km), $f$ is the flattening (0.003352811) and $m$ is the gravity ratio (0.003449787). This is implemented in `ropp_utils/geodesy/r_eff.f90`.

In the Invert code (`Lib/Earth.f90`), an effective Earth radius is given by

$$R_s(\phi) = \frac{(g_{sfc}/g_e)R_e}{1 + f + m + (-3f + 5m/2)\sin^2\phi} \tag{1.8}$$

where $R_e$ = 6378.1353 km is the equatorial semiaxis.

Figure 1.2 shows the difference between Earth radius values computed using Equations 1.7 and 1.8. Note that Equation 1.8 uses the ratio $g_{sfc}/g_e$, which is consistent between ROPP and Invert implementations. Figure 1.2 shows the discrepancy between $R_s(\phi)$ values increasing up to 150 m at the pole.

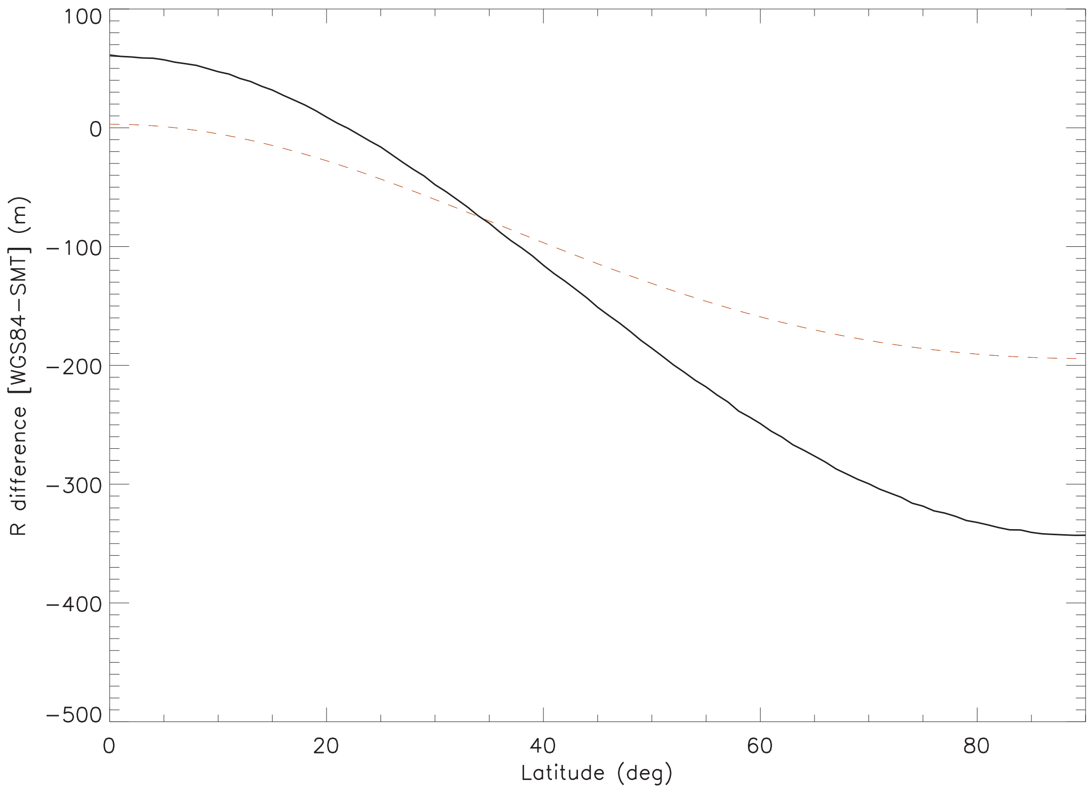



**Figure 1.2:** Black: Difference between Earth radius computed using Somigliana's equation (Equation 1.7) for WGS-84 ellipsoid and the values derived using the Smithsonian Meteorological Tables (see GSR-02). Red: Difference between Earth radius computed using Equation 1.7 and Equation 1.8.

- Is Equation 1.8 'more accurate' (i.e. higher order approximation) than Equation 1.7
- Where does Equation 1.8 originate from? Is it derived from a Taylor expansion of the Somagliana Equation, just to higher order?

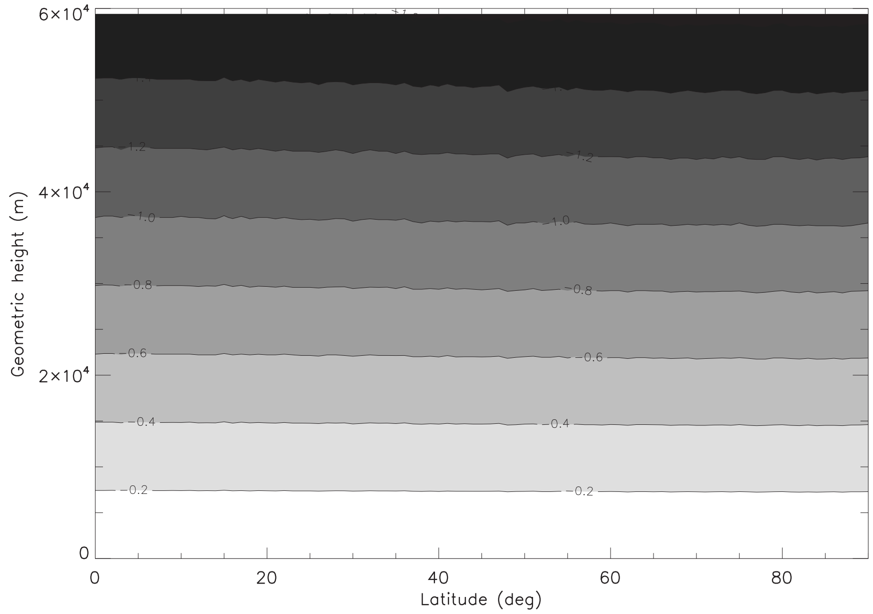
## 1.3 Geopotential height

In both ROPP and Invert codes, the geopotential height ($Z$) is converted from geometric height ($H$) by the relation

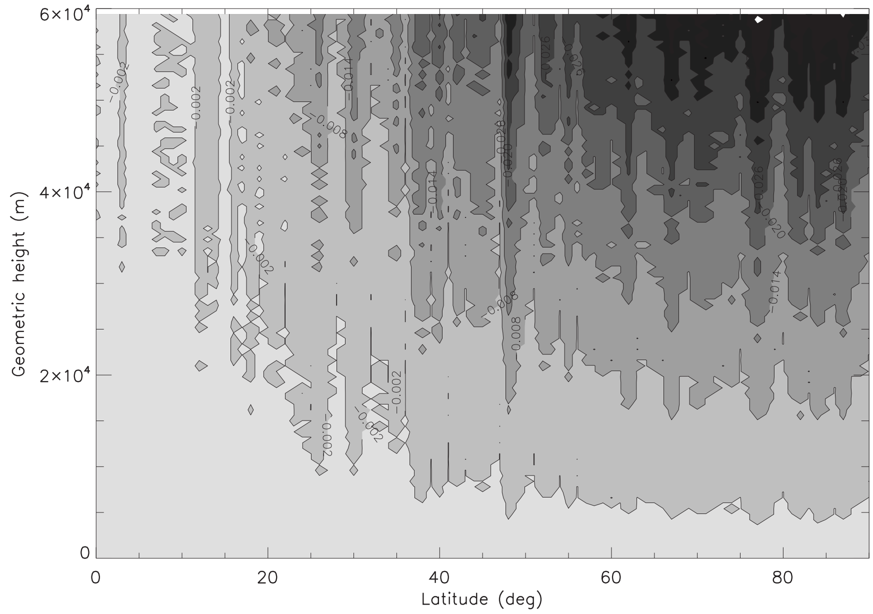
$$Z = \frac{g_s(\phi)}{g_{av}} \frac{R_s(\phi)H}{R_s(\phi) + H} \tag{1.9}$$

where $g_{av}$ = 9.80665 ms$^{-2}$ is the average gravity acceleration.

The impact of the differences in $g_s(\phi)$ and $R_s(\phi)$ resulting from the different equations used in ROPP and Invert is shown in Figure 1.3. The largest contribution to differences between ROPP and Invert conversions to geopotential height results from the discrepancy between $g_s(\phi)$. Figure 1.3(a) shows this results in differences increasing up to 1.5 m at 60 km (independent of $\phi$). Figure 1.3(b) shows the difference between ROPP and Invert geopotential height computed using the same constant value for the equatorial gravity $g_e$ in both cases. This results in differences within 5 cm over all latitude ranges. The differences in $R_s(\phi)$ shown in Figure 1.2 therefore have negligible impact on the geopotential height results, but differences in $g_s(\phi)$ (resulting from different $g_e$) are evident when comparing results from the two software packages.




(a)




(b)

**Figure 1.3:** Difference between geopotential height computed using values of $g(\phi)$ and $R(\phi)$ based on Somigliana's equation as implemented in ROPP and Invert. (a) Difference between current ROPP and Invert implementations. (b) Difference between ROPP and Invert using a 'constant' value for $g_e$ rather than that computed from Equation 1.3.

# Bibliography

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