1 Comparison of the refractivity geopotentials and impact heights in `ropp_fm_bg2ro_1d`

Fig 1.1 compares the refrac levels and impact heights for three runs of `ropp_fm_bg2ro_1d`. The background (ECMWF) profile passes though a strongly capped marine boundary layer at the edge of the Azores high. (It is the tenth profile in the '55-profile WOP test dataset'.)

The green line shows the results of running the forward model through 7501 uniformly spaced (Δz = 10 m) levels between 0 and 75 km. This serves as the 'exact' result. (Its kinks reflect those in the relatively coarse 91L background, which are located at the dotted black lines.) Three curves are shown: the refractivity geopotential z, the refractivity N and the resulting impact height $a - R_c = (1 + 10^{-6} N(z))r - R_c$, where $r = h(z) + R_c + u$, u being the undulation, R_c being the radius of curvature and h being the geometric height. The sharp drop in refractivity across the top of the boundary layer at around 700 m is clear. A consequence of this is that the impact heights are necessarily non-monotonic in this region. Formally, $rn(r)$ is a decreasing function of r if $n'(r) < -n/r \approx -1.57 \times 10^{-6}$ m^{-1}. This ('ducting') condition is met between 500 and 800 m for this profile.

The blue diamonds show the results of running the default option of `ropp_fm_bg2ro_1d`, namely 300 uniformly spaced (Δz = 200 m) levels between 200 m and 60 km. The non-monotonicity of the impact heights also holds at this resolution.

The red circles show the results of running with the -247L option. In this case, the refractivity geopotentials $z(r)$are found by iteratively inverting the above equation for the impact heights. Three issues are clear. Firstly, we need to invent refractivities below the surface of the model (z_{sfc}) in order to generate z values that correspond with the lowest impact heights. In this case, the first two refractivities (plotted as solid red circles) are fictitious. Nonetheless, they and the resulting refractivity geopotentials z_1 and z_2 look reasonable. (The extrapolation is done by treating N as a function of i rather than as a function of z, in order to avoid extrapolating excessively steep physical refractivity gradients over large depths, such as those that occur in Antarctica, where temperature gradients of over 20 K/km need to be extrapolated over thousands of metres to reach the lowest impact height.) Secondly, the defining equation $rn(r) = a$ conspires against the ROPP user interested in sharp refractivity gradients (as occur in this profile), because the quasi-uniform separation of the impact heights means that a large decrease in refractivity is accompanied by a large increase in geopotential height: $rn(r) = a$ implies $n\Delta r + r\Delta n = \Delta a \approx$ const. Vertical resolution is therefore coarsened in precisely the regions where it needs to be finest. Users concerned with dN/dz are best advised to use high uniform vertical resolution options of `ropp_fm_bg2ro_1d`. Thirdly, close study of Fig 1.1 reveals that there are three possible solutions of $rn(r) = a$ at the impact height in the boundary layer at $a - R_c$ = 2690 m (red dashed line): one at $z \approx$ 500 m, one at $z \approx$ 600 m, and the one selected by the iteration scheme, $z \approx$ 920 m. This non-uniqueness is a consequence of the non-monotonicity of $a(z)$. Which of the three solutions is 'right' is undecidable, and, as a matter of fact, neither z = 500 m nor z = 600 m resolves dN/dz any better than choosing z = 920 m. But the ambiguity reveals another risk in using coarsely spaced impact heights in the presence of steep refractivity gradients.

For this profile, the check within `ropp_fm_abel` that successive source nr values increase by at least 10 m means that level 11 (at z=780 m, shown as z_{lower} in dashed black) is the lowest one that can be used in the calculation of bending angles. In addition, $-dN/d(nr)$ is limited to be less than 0.157 N-units m^{-1} between levels 9 and 13 (shaded region). In the 247L case, this means that the lowest 5 bending angles are missing, as is clear from the figure. (And the 'limited' $-dN/d(nr)$ values only apply to the sixth bending angle.) In the 300L case, only the first bending angle is missing, because all the other levels have impact heights greater than $nr(11)$, *even if their geopotentials are below* z_{lower}. (This is another effect of the non-monotonicity of $a(z)$.) The Abel integral that generates the bending angles starts from model level 11 in all these cases, so the 'limited' $-dN/d(nr)$ values apply to the 2nd, 3rd, 4th and 5th bending angles.

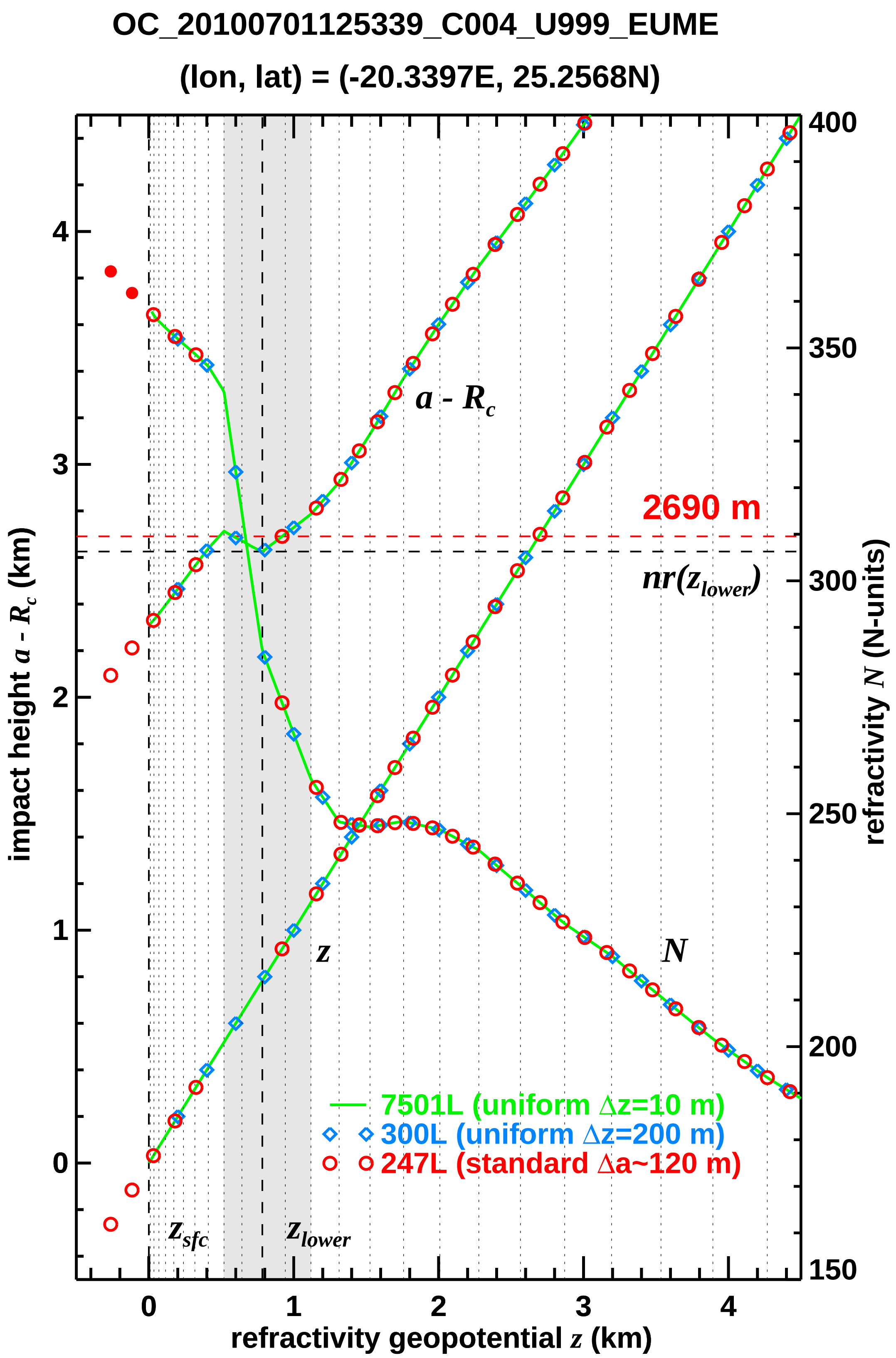

Figure 1.1: Geopotentials z, refractivities N and impact heights $a - R_c$, for three runs of `ropp_fm_bg2ro_1d`: uniform high resolution (10 m) refractivity levels (green line); uniform default resolution (200 m) refractivity levels (blue diamonds); quasi-uniform (≈120 m) default impact heights (red circles).