Chapter 2
Coordinate and Time Systems

Satellites orbit around the Earth or travel in the planet system of the sun. They are generally observed from the Earth. To describe the orbits of the satellites (positions and velocities), suitable coordinate and time systems have to be defined.

2.1 Geocentric Earth-Fixed Coordinate Systems

It is convenient to use the Earth-Centred Earth-Fixed (ECEF) coordinate system to describe the location of a station on the Earth's surface. The ECEF coordinate system is a right-handed Cartesian system (x, y, z). Its origin and the Earth's centre of mass coincide, while its z-axis and the mean rotational axis of the Earth coincide; the x-axis points to the mean Greenwich meridian, while the y-axis is directed to complete a right-handed system (Fig. 2.1). In other words, the z-axis points to a mean pole of the Earth's rotation. Such a mean pole, defined by international convention, is called the Conventional International Origin (CIO). The xy-plane is called the mean equatorial plane, and the xz-plane is called the mean zero-meridian.

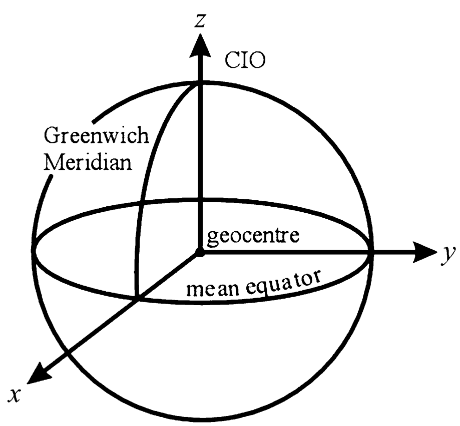

Fig. 2.1 Earth-Centred Earth-Fixed coordinates

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The ECEF coordinate system is also known as the Conventional Terrestrial System (CTS). The mean rotational axis and mean zero-meridian used here are necessary. The true rotational axis of the Earth changes its direction all the time with respect to the Earth's body. If such a pole is used to define a coordinate system, then the coordinates of the station would also change all the time. Because the survey is made in our true world, it is obvious that the polar motion has to be taken into account and will be discussed later.

The ECEF coordinate system can, of course, be represented by a spherical coordinate system (r, ϕ, λ), where r is the radius of the point (x, y, z), and ϕ and λ are the geocentric latitude and longitude, respectively (Fig. 2.2). λ is counted eastward from the zero-meridian. The relationship between (x, y, z) and (r, ϕ, λ) is obvious:

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} r\cos\phi\cos\lambda \\ r\cos\phi\sin\lambda \\ r\sin\phi \end{pmatrix} \quad \text{or} \quad \begin{cases} r = \sqrt{x^2+y^2+z^2}, \\ \tan\lambda = y/x, \\ \tan\phi = z/\sqrt{x^2+y^2}. \end{cases} \tag{2.1}$$

An ellipsoidal coordinate system (φ, λ, h) may also be defined on the basis of the ECEF coordinates; however, geometrically, two additional parameters are needed to define the shape of the ellipsoid (Fig. 2.3). φ, λ and h are geodetic latitude, longitude and height, respectively. The ellipsoidal surface is a rotational ellipse. The ellipsoidal system is also called the geodetic coordinate system. Geocentric longitude and geodetic longitude are identical. The two geometric parameters could be the semi-major radius (denoted by a) and the semi-minor radius (denoted by b) of the rotating ellipse, or the semi-major radius and the flattening (denoted by f) of the ellipsoid. They are equivalent sets of parameters. The relationship between (x,y,z) and (φ, λ, h) is (see, e.g., Torge, 1991):

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} (N+h)\cos\varphi\cos\lambda \\ (N+h)\cos\varphi\sin\lambda \\ (N(1-e^2)+h)\sin\varphi \end{pmatrix} \tag{2.2}$$

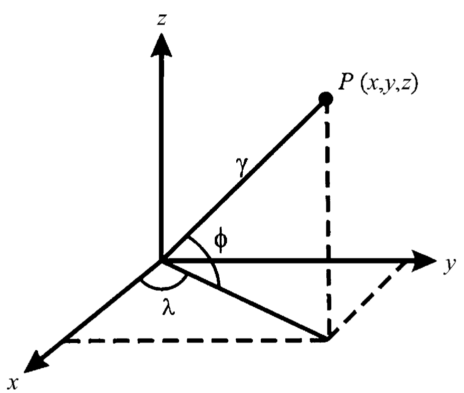


Fig. 2.2 Cartesian and spherical coordinates

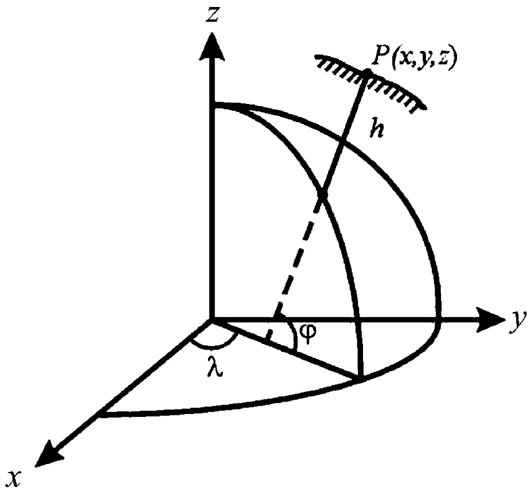


Fig. 2.3 Ellipsoidal coordinate system

or

$$\begin{cases} \tan\varphi = \dfrac{z}{\sqrt{x^2+y^2}}\left(1-e^2\frac{N}{N+h}\right)^{-1}, \\ \tan\lambda = \dfrac{y}{x}, \\ h = \dfrac{\sqrt{x^2+y^2}}{\cos\varphi} - N, \end{cases} \tag{2.3}$$

where

$$N = \frac{a}{\sqrt{1-e^2\sin^2\varphi}}. \tag{2.4}$$

N is the radius of curvature in the prime vertical, and e is the first eccentricity. The geometric meaning of N is shown in Fig. 2.4. In (2.3), the φ and h have to be solved by iteration; however, the iteration process converges quickly, since $h << N$. The flattening and the first eccentricity are defined as

$$f = \frac{a-b}{a} \quad \text{and} \quad e = \frac{\sqrt{a^2-b^2}}{a}. \tag{2.5}$$

In cases where $\varphi = \pm 90°$ or h is very large, the iteration formulas of (2.3) could be instable. Alternatively, using

$$\mathrm{c}\tan\varphi = \frac{\sqrt{x^2+y^2}}{z+\Delta z} \quad \text{and} \quad \Delta z = e^2 N\sin\varphi = \frac{ae^2\sin\varphi}{\sqrt{1-e^2\sin^2\varphi}},$$

may lead to a stably iterated result of φ (see Lelgemann, 2002). Δz and e^2N are the lengths of $\overline{OB}$ and $\overline{AB}$ (see Fig. 2.4), respectively. The geodetic height h can be obtained using Δz, i.e.,

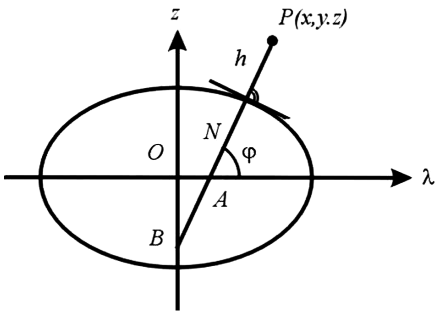


Fig. 2.4 Radius of curvature in the prime vertical

$$h = \sqrt{x^2 + y^2 + (z + \Delta z)^2} - N.$$

The two geometric parameters used in the World Geodetic System 1984 (WGS-84) are ($a = 6378137\,\mathrm{m}$, $f = 1/298.2572236$). In International Terrestrial Reference Frame 1996 (ITRF-96), the two parameters are ($a = 6378136.49\,\mathrm{m}$, $f = 1/298.25645$). ITRF uses the International Earth Rotation Service (IERS) Conventions (see McCarthy, 1996). In the PZ-90 (Parameters of the Earth Year 1990) coordinate system of GLONASS, the two parameters are ($a = 6378136\,\mathrm{m}$, $f = 1/298.2578393$).

The relation between the geocentric and geodetic latitude ϕ and φ (see (2.1) and (2.3)) may be given by

$$\tan\phi = \left(1 - e^2 \frac{N}{N+h}\right) \tan\varphi. \tag{2.6}$$

2.2 Coordinate System Transformations

Any Cartesian coordinate system can be transformed to another Cartesian coordinate system through three successive rotations if their origins are the same and if they are both right-handed or left-handed coordinate systems. These three rotational matrices are

$$R_1(\alpha) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & \sin\alpha \\ 0 & -\sin\alpha & \cos\alpha \end{pmatrix},$$

$$R_2(\alpha) = \begin{pmatrix} \cos\alpha & 0 & -\sin\alpha \\ 0 & 1 & 0 \\ \sin\alpha & 0 & \cos\alpha \end{pmatrix}, \tag{2.7}$$

$$R_3(\alpha) = \begin{pmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

where α is the rotating angle, which has a positive sign for a counter-clockwise rotation as viewed from the positive axis to the origin. R_1, R_2, and R_3 are called the rotating matrix around the x, y, and z-axis, respectively. For any rotational matrix R, there are properties of $R^{-1}(\alpha) = R^T(\alpha)$ and $R^{-1}(\alpha) = R(-\alpha)$; that is, the rotational matrix is an orthogonal one, where R^{-1} and R^T are the inverse and transpose of the matrix R.

For two Cartesian coordinate systems with different origins and different length units, the general transformation can be given in vector (matrix) form as

$$X_n = X_0 + \mu R X_{\text{old}} \tag{2.8}$$

or

$$\begin{pmatrix} x_n \\ y_n \\ z_n \end{pmatrix} = \begin{pmatrix} x_0 \\ y_0 \\ z_0 \end{pmatrix} + \mu R \begin{pmatrix} x_{\text{old}} \\ y_{\text{old}} \\ z_{\text{old}} \end{pmatrix},$$

where μ is the scale factor (or the ratio of the two length units), and R is a transformation matrix that can be formed by three suitably successive rotations. x_n and x_{old} denote the new and old coordinates, respectively; x_0 denotes the translation vector and is the coordinate vector of the origin of the old coordinate system in the new one.

If rotational angle α is very small, then one has $\sin\alpha \approx \alpha$ and $\cos\alpha \approx 1$. In such a case, the rotational matrix can be simplified. If the three rotational angles α_1, α_2, α_3 in R of (2.8) are very small, then R can be written as

$$R = \begin{pmatrix} 1 & \alpha_3 & -\alpha_2 \\ -\alpha_3 & 1 & \alpha_1 \\ \alpha_2 & -\alpha_1 & 1 \end{pmatrix}, \tag{2.9}$$

where α_1, α_2, α_3 are small rotating angles around the x, y and z-axis, respectively (see, e.g., Lelgemann and Xu, 1991). Using the simplified R, the transformation (2.8) is called the Helmert transformation.

As an example, the transformation from WGS-84 to ITRF-90 (McCarthy, 1996) is given by:

$$\begin{pmatrix} x_{\text{ITRF-90}} \\ y_{\text{ITRF-90}} \\ z_{\text{ITRF-90}} \end{pmatrix} = \begin{pmatrix} 0.060 \\ -0.517 \\ -0.223 \end{pmatrix} + \mu \begin{pmatrix} 1 & -0.0070'' & -0.0003'' \\ 0.0070'' & 1 & -0.0183'' \\ 0.0003'' & 0.0183'' & 1 \end{pmatrix} \begin{pmatrix} x_{\text{WGS-84}} \\ y_{\text{WGS-84}} \\ z_{\text{WGS-84}} \end{pmatrix},$$

where $\mu = 0.999999989$, and the translation vector has the unit of meter.

The transformation between two coordinate systems can be generally represented by (2.8), where the scale factor $\mu = 1$ (i.e., the units of length used nowadays are the same). A formula of velocity transformation between different coordinate systems can be obtained by differentiating (2.8) with respect to the time.

2.3 Local Coordinate System

The local left-handed Cartesian coordinate system (x',y',z') can be defined by placing the origin to the local point $P_1(x_1,y_1,z_1)$, whose z'-axis is pointed to the vertical, x'-axis is directed to the north, and y' is pointed to the east (see Fig. 2.5). The $x'y'$-plane is called the horizontal plane; the vertical is defined perpendicular to the ellipsoid. Such a coordinate system is also called a local horizontal coordinate system. For any point P_2, whose coordinates in the global and local coordinate system are (x_2,y_2,z_2) and (x',y',z'), respectively, one has relations of

$$\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = d \begin{pmatrix} \cos A \sin Z \\ \sin A \sin Z \\ \cos Z \end{pmatrix}, \quad \text{and} \quad \begin{pmatrix} d = \sqrt{x'^2 + y'^2 + z'^2} \\ \tan A = y'/x' \\ \cos Z = z'/d \end{pmatrix}, \tag{2.10}$$

where A is the azimuth, Z is the zenith distance and d is the radius of the P_2 in the local system. A is measured from the north clockwise; Z is the angle between the vertical and the radius d.

The local coordinate system (x',y',z') can indeed be obtained by two successive rotations of the global coordinate system (x,y,z) by $R_2(90° - \varphi)R_3(\lambda)$ and then by changing the x-axis to a right-handed system. In other words, the global system has to be rotated around the z-axis with angle λ, then around the y-axis with angle $90° - \varphi$, and then change the sign of the x-axis. The total transformation matrix R is then

$$R = \begin{pmatrix} -\sin\varphi\cos\lambda & -\sin\varphi\sin\lambda & \cos\varphi \\ -\sin\lambda & \cos\lambda & 0 \\ \cos\varphi\cos\lambda & \cos\varphi\sin\lambda & \sin\varphi \end{pmatrix}, \tag{2.11}$$

and there are

$$X_{\text{local}} = RX_{\text{global}} \quad \text{and} \quad X_{\text{global}} = R^T X_{\text{local}}, \tag{2.12}$$

where X_{local} and X_{global} are the same vector represented in local and global coordinate systems. (φ, λ) are the geodetic latitude and longitude of the local point.

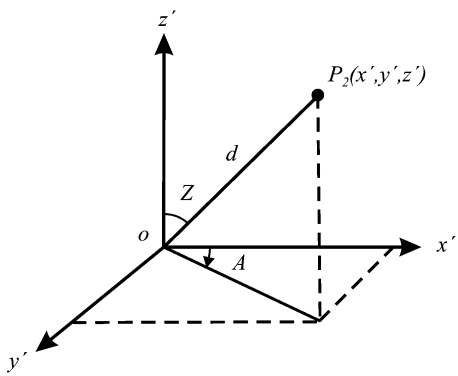


Fig. 2.5 Astronomical coordinate system

If the vertical direction is defined as the plumb line of the gravitational field at the local point, then such a local coordinate system is called an astronomic horizontal system (its x'-axis is pointed to the north, left-handed system). The plumb line of gravity g and the vertical line of the ellipsoid at the point p are generally not coinciding with each other; however, the difference is very small. The difference is omitted in GPS practice.

Combining (2.10) and (2.12), the zenith angle and azimuth of a point P_2 (satellite) related to the station P_1 can be directly computed by using the global coordinates of the two points by

$$\cos Z = \frac{z'}{d} \quad \text{and} \quad \tan A = \frac{y'}{x'}, \tag{2.13}$$

where

$$\begin{aligned}
d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}, \\
x' &= -(x_2 - x_1)\sin\varphi\cos\lambda - (y_2 - y_1)\sin\varphi\sin\lambda + (z_2 - z_1)\cos\varphi, \\
y' &= -(x_2 - x_1)\sin\lambda + (y_2 - y_1)\cos\lambda
\end{aligned}$$

and

$$z' = (x_2 - x_1)\cos\varphi\cos\lambda + (y_2 - y_1)\cos\varphi\sin\lambda + (z_2 - z_1)\sin\varphi.$$

2.4 Earth-Centred Inertial Coordinate System

To describe the motion of the GPS satellites, an inertial coordinate system has to be defined. The motion of the satellites follows the Newtonian mechanics, and the Newtonian mechanics is valid and expressed in an inertial coordinate system. For various reasons, the Conventional Celestial Reference Frame (CRF) is suitable for our purpose. The xy-plane of the CRF is the plane of the Earth's equator; the coordinates are celestial longitude, measured eastward along the equator from the vernal equinox, and celestial latitude. The vernal equinox is a crossover point of the ecliptic and the equator. So the right-handed Earth-centred inertial (ECI) system uses the Earth centre as the origin, CIO (Conventional International Origin) as the z-axis, and its x-axis is directed to the equinox of J2000.0 (Julian Date of 12h 1st January 2000). Such a coordinate system is also called equatorial coordinates of date. Because of the motion (acceleration) of the Earth's centre, ECI is indeed a quasi-inertial system, and the general relativistic effects have to be taken into account in this system. The system moves around the sun, however, without rotating with respect to the CIO. This system is also called the Earth-centred space-fixed (ECSF) coordinate system.

An excellent figure has been given by Torge (1991) to illustrate the motion of the Earth's pole with respect to the ecliptic pole (see Fig. 2.6). The Earth's flattening, combined with the obliquity of the ecliptic, results in a slow turning of the equator on the ecliptic due to the differential gravitational effect of the moon and the sun. The slow circular motion with a period of about 26000 years is called precession, and the other quicker motion with periods ranging from 14 days to 18.6 years is

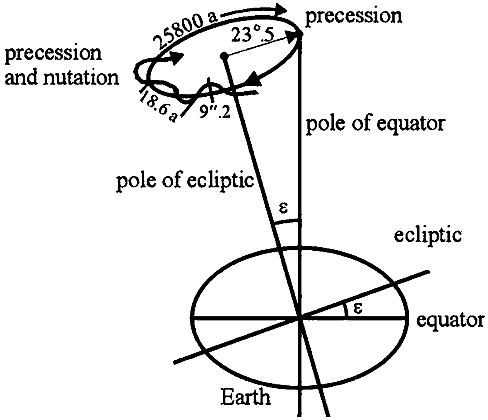


Fig. 2.6 Precession and nutation

called nutation. Taking the precession and nutation into account, the Earth's mean pole (related to the mean equator) is transformed to the Earth's true pole (related to the true equator). The x-axis of the ECI is pointed to the vernal equinox of date.

The angle of the Earth's rotation from the equinox of date to the Greenwich meridian is called Greenwich Apparent Sidereal Time (GAST). Taking GAST into account (called the Earth's rotation), the ECI of date is transformed to the true equatorial coordinate system. The difference between the true equatorial system and the ECEF system is the polar motion. So we have transformed the ECI system in a geometric way to the ECEF system. Such a transformation process can be written as

$$X_{\mathrm{ECEF}} = R_{\mathrm{M}} R_{\mathrm{S}} R_{\mathrm{N}} R_{\mathrm{P}} X_{\mathrm{ECI}}, \tag{2.14}$$

where R_{P} is the precession matrix, R_{N} is the nutation matrix, R_{S} is the Earth rotation matrix, R_{M} is the polar motion matrix, X is the coordinate vector, and indices ECEF and ECI denote the related coordinate systems.

Precession

The precession matrix consists of three successive rotational matrices, i.e. (see, e.g., Hofmann-Wellenhof et al., 1997/2001; Leick, 1995/2004; McCarthy, 1996),

$$\begin{aligned} R_{\mathrm{P}} &= R_3(-z)R_2(\theta)R_3(-\zeta) \\ &= \begin{pmatrix} \cos z\cos\theta\cos\zeta - \sin z\sin\zeta & -\cos z\cos\theta\sin\zeta - \sin z\cos\zeta & -\cos z\sin\theta \\ \sin z\cos\theta\cos\zeta + \cos z\sin\zeta & -\sin z\cos\theta\sin\zeta + \cos z\cos\zeta & -\sin z\sin\theta \\ \sin\theta\cos\zeta & -\sin\theta\sin\zeta & \cos\theta \end{pmatrix}, \end{aligned} \tag{2.15}$$

where z, θ, ζ are precession parameters and

$$\begin{aligned} z &= 2306.''2181T + 1.''09468T^2 + 0.''018203T^3, \\ \theta &= 2004.''3109T - 0.''42665T^2 - 0.''041833T^3 \end{aligned} \tag{2.16}$$

and

$$\zeta = 2306.''2181T + 0.''30188T^2 + 0.''017998T^3,$$

where T is the measuring time in Julian centuries (36525 days) counted from J2000.0 (see Sect. 2.8 time systems).

Nutation

The nutation matrix consists of three successive rotational matrices, i.e. (see, e.g., Hofmann-Wellenhof et al., 1997/2001; Leick, 1995/2004; McCarthy, 1996)

$$\begin{aligned} R_{\mathrm{N}} &= R_1(-\varepsilon - \Delta\varepsilon)R_3(-\Delta\psi)R_1(\varepsilon) \\ &= \begin{pmatrix} \cos\Delta\psi & -\sin\Delta\psi\cos\varepsilon & -\sin\Delta\psi\sin\varepsilon \\ \sin\Delta\psi\cos\varepsilon_t & \cos\Delta\psi\cos\varepsilon_t\cos\varepsilon + \sin\varepsilon_t\sin\varepsilon & \cos\Delta\psi\cos\varepsilon_t\sin\varepsilon - \sin\varepsilon_t\cos\varepsilon \\ \sin\Delta\psi\sin\varepsilon_t & \cos\Delta\psi\sin\varepsilon_t\cos\varepsilon - \cos\varepsilon_t\sin\varepsilon & \cos\Delta\psi\sin\varepsilon_t\sin\varepsilon + \cos\varepsilon_t\cos\varepsilon \end{pmatrix} \\ &\approx \begin{pmatrix} 1 & -\Delta\psi\cos\varepsilon & -\Delta\psi\sin\varepsilon \\ \Delta\psi\cos\varepsilon_t & 1 & -\Delta\varepsilon \\ \Delta\psi\sin\varepsilon_t & \Delta\varepsilon & 1 \end{pmatrix}, \end{aligned} \tag{2.17}$$

where ε is the mean obliquity of the ecliptic angle of date, $\Delta\psi$ and $\Delta\varepsilon$ are nutation angles in longitude and obliquity, $\varepsilon_t = \varepsilon + \Delta\varepsilon$, and

$$\varepsilon = 84381.''448 - 46.''8150T - 0.''00059T^2 + 0.''001813T^3. \tag{2.18}$$

The approximation is made by letting $\cos\Delta\psi = 1$ and $\sin\Delta\psi = \Delta\psi$ for very small $\Delta\psi$. For precise purposes, the exact rotation matrix shall be used. The nutation parameters $\Delta\psi$ and $\Delta\varepsilon$ can be computed using the International Astronomical Union (IAU) theory or IERS theory:

$$\Delta\Psi = \sum_{i=1}^{106} (A_i + A_i'T)\sin\beta,$$
$$\Delta\varepsilon = \sum_{i=1}^{106} (B_i + B_i'T)\cos\beta,$$

or

$$\Delta\Psi = \sum_{i=1}^{263} (A_i + A_i'T)\sin\beta + A_i''\cos\beta,$$
$$\Delta\varepsilon = \sum_{i=1}^{263} (B_i + B_i'T)\cos\beta + B_i''\cos\beta,$$

where argument

$$\beta = N_{1i}l + N_{2i}l' + N_{3i}F + N_{4i}D + N_{5i}\Omega,$$

where l is the mean anomaly of the moon, l' is the mean anomaly of the sun, $F = L - \Omega, D$ is the mean elongation of the moon from the sun, Ω is the mean longitude of the ascending node of the moon, and L is the mean longitude of the moon. The formulas of l, l', F, D, and Ω, are given in Sect. 7.8. The coefficient values of N_{1i}, N_{2i}, N_{3i}, N_{4i}, N_{5i}, A_i, B_i, A'_i, B'_i, A''_i, and B''_i can be found in, e.g., McCarthy (1996). The updated formulas and tables can be found in updated IERS conventions. For convenience, the coefficients of the IAU 1980 nutation model are given in Appendix 1.

Earth Rotation

The Earth rotation matrix can be represented as

$$R_S = R_3(\mathrm{GAST}), \tag{2.19}$$

where GAST is Greenwich Apparent Sidereal Time and

$$\mathrm{GAST} = \mathrm{GMST} + \Delta\Psi \cos\varepsilon + 0.''00264 \sin\Omega + 0.''000063 \sin 2\Omega, \tag{2.20}$$

where GMST is Greenwich Mean Sidereal Time. Ω is the mean longitude of the ascending node of the moon; the second term on the right-hand side is the nutation of the equinox. Furthermore,

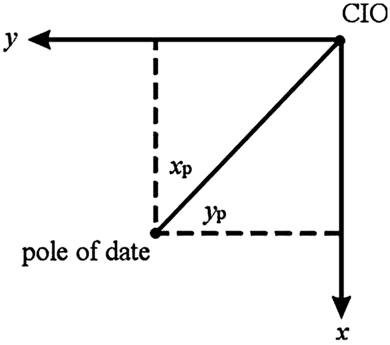
$$\begin{aligned} \mathrm{GMST} &= \mathrm{GMST}_0 + \alpha \mathrm{UT1}, \\ \mathrm{GMST}_0 &= 6 \times 3600.''0 + 41 \times 60.''0 + 50.''54841 \\ &\quad + 8\,640\,184.''812866 T_0 + 0.''093104 T_0^2 - 6.''2 \times 10^{-6} T_0^3, \\ \alpha &= 1.002737909350795 + 5.9006 \times 10^{-11} T_0 - 5.9 \times 10^{-15} T_0^2, \end{aligned} \tag{2.21}$$

where GMST_0 is Greenwich Mean Sidereal Time at midnight on the day of interest. α is the rate of change. UT1 is the polar motion corrected Universal Time (see Sect. 2.8). T_0 is the measuring time in Julian centuries (36525 days) counted from J2000.0 to 0h UT1 of the measuring day. By computing GMST, UT1 is used (see Sect. 2.8).

Polar Motion

As shown in Fig. 2.7, the polar motion is defined as the angles between the pole of date and the CIO pole. The polar motion coordinate system is defined by xy-plane coordinates, whose x-axis is pointed to the south and is coincided to the mean Greenwich meridian, and whose y-axis is pointed to the west. x_p and y_p are the angles of the pole of date, so the rotation matrix of polar motion can be represented as

Fig. 2.7 Polar motion



$$R_M = R_2(-x_p)R_1(-y_p) = \begin{pmatrix} \cos x_p & \sin x_p \sin y_p & \sin x_p \cos y_p \\ 0 & \cos y_p & -\sin y_p \\ -\sin x_p & \cos x_p \sin y_p & \cos x_p \cos y_p \end{pmatrix}$$
$$\approx \begin{pmatrix} 1 & 0 & x_p \\ 0 & 1 & -y_p \\ -x_p & y_p & 1 \end{pmatrix}. \tag{2.22}$$

The IERS determined x_p and y_p can be obtained from the home pages of IERS.

2.5 IAU 2000 Framework

At its 2000 General Assembly, the International Astronomical Union (IAU) adopted a set of resolutions that provide a consistent framework for defining the barycentric and geocentric celestial reference systems (Petit, 2002). The consequence of the resolution is that the coordinate transformation from celestial reference system (CRS, i.e., the ECI system) to the terrestrial reference system (TRS, i.e., the ECEF system) has the form

$$X_{ECEF} = R_M R_S R_{NP} X_{ECI}, \tag{2.23}$$

where R_{NP} is the precession-nutation matrix, R_S is the Earth rotation matrix, R_M is the polar motion matrix, X is the coordinate vector, and indices ECEF and ECI denote the related coordinate systems. The rotation matrices are functions of time T which is defined (see McCarthy and Petit, 2003) by

$$T = (\mathrm{TT} - 2000\text{January 1d 12h TT}) \text{ in days/36525}, \tag{2.24}$$

where TT is the Terrestrial Time (for details see Sect. 2.8) and

$$\begin{aligned} R_M &= R_2(-x_p)R_1(-y_p)R_3(s'), \\ R_S &= R_3(\vartheta) \end{aligned} \tag{2.25}$$

and

$$R_{\mathrm{NP}} = R_3(-s)R_3(-E)R_2(d)R_3(E),$$

where x_{p} and y_{p} are the angles of the pole of date (or polar coordinates of the Celestial Intermediate Pole (CIP) in TRS), and s' is a function of x_{p} and y_{p}:

$$s' = \frac{1}{2}\int_{T0}^{T} (x_{\mathrm{p}}\dot{y}_{\mathrm{p}} - \dot{x}_{\mathrm{p}}y_{\mathrm{p}})\mathrm{d}t$$

or approximately (see McCarthy and Capitaine, 2002)

$$s' = (-47\mu as)T, \tag{2.26}$$

where T is time in Julian Century counted from J2000.0 and

$$\vartheta = 2\pi(0.7790572732640 + 1.00273781191135448\,T_u), \tag{2.27}$$

where T_u = (Julian UT1 date – 2451545.0) and $\mathrm{UT1} = \mathrm{UTC} + (\mathrm{UT1} - \mathrm{UTC}) \cdot (\mathrm{UT1} - \mathrm{UTC})$ is published by the IERS.

E and d being such that the coordinates of the CIP in the CRS are

$$\begin{aligned} X &= \sin d \cos E, \\ Y &= \sin d \sin E, \\ Z &= \cos d. \end{aligned} \tag{2.28}$$

Equivalently R_{NP} can be given by

$$R_{\mathrm{NP}} = R_3(-s) \cdot \begin{pmatrix} 1 - aX^2 & -aXY & X \\ -aXY & 1 - aY^2 & Y \\ -X & -Y & 1 - a(X^2 + Y^2) \end{pmatrix}^{-1}, \tag{2.29}$$

where

$$a = \frac{1}{1 + \cos d} \approx \frac{1}{2} + \frac{1}{8}(X^2 + Y^2). \tag{2.30}$$

The developments of X and Y can be found on the website of the IERS Conventions and have the following form (in mas: microarcsecond) (Capitaine, 2002)

$$\begin{aligned} X = {} & -16616.99'' + 2004191742.88''T - 427219.05''T^2 \\ & - 198620.54''T^3 - 46.05''T^4 + 5.98''T^5 \\ & + \textstyle\sum_i [(a_{s,0})_i \sin\beta + (a_{c,0})_i \cos\beta] \\ & + \textstyle\sum_i [(a_{s,1})_i T \sin\beta + (a_{c,1})_i T \cos\beta] \\ & + \textstyle\sum_i [(a_{s,2})_i T^2 \sin\beta + (a_{c,2})_i T^2 \cos\beta] + \cdots, \end{aligned} \tag{2.31}$$

$$\begin{aligned} Y = & -6950.78'' - 25381.99''T - 22407250.99''T^2 \\ & + 1842.28''T^3 - 1113.06''T^4 + 0.99''T^5 \\ & + \textstyle\sum_i [(b_{s,0})_i \sin\beta + (b_{c,0})_i \cos\beta] \\ & + \textstyle\sum_i [(b_{s,1})_i T \sin\beta + (b_{c,1})_i T \cos\beta] \\ & + \textstyle\sum_i [(b_{s,2})_i T^2 \sin\beta + (b_{c,2})_i T^2 \cos\beta] + \cdots \end{aligned} \tag{2.32}$$

s in (2.29) is the accumulated rotation, between the reference epoch and the date T, of CEO on the true equator due to the celestial motion of CIP, and can be expressed as

$$s(T) = -\frac{1}{2}[X(T)Y(T) - X(T_0)Y(T_0)] + \int_{T_0}^{T} \dot{X}Y\mathrm{d}t - (\sigma_0 N_0 - \textstyle\sum_0 N_0),$$

where σ_0 and $\sum_0$ are the positions of CEO at J2000.0 and the x-origin of CRS, respectively and N_0 is the ascending node at J2000.0 in the equator of CRS. In above equation, terms$(T) + \frac{1}{2}[X(T)Y(T)]$ can be expressed as (in mas):

$$\begin{aligned} s + XY/2 = & \; 94.0 + 3808.35T - 119.94T^2 \\ & - 72574.09T^3 + 27.70T^4 + 15.61T^5 \\ & + \textstyle\sum_i [(c_{s,0})_i \sin\beta + (c_{c,0})_i \cos\beta] \\ & + \textstyle\sum_i [(c_{s,1})_i T \sin\beta + (c_{c,1})_i T \cos\beta] \\ & + \textstyle\sum_i [(c_{s,2})_i T^2 \sin\beta + (c_{c,2})_i T^2 \cos\beta] + \cdots \end{aligned} \tag{2.33}$$

In (2.31), (2.32) and (2.33), coefficients $(a_{s,j})_i, (a_{c,j})_i, (b_{s,j})_i, (b_{c,j})_i$ and $(c_{s,j})_i, (c_{c,j})_i$ can be extracted from table5.2a, table5.2b and table5.2c (available at ftp://tai.bipm.org/iers/conv2003/chapter5/). β is the combination of the fundamental arguments of nutation theory

$$\beta = \sum_{j=1}^{14} N_j F_j. \tag{2.34}$$

The first five F_j are the Delaunary variables l, l', F, D, Ω (given in Sect. 7.8); the amplitudes of sines and cosines β can be derived from the amplitudes of the precession and nutation series (see McCarthy and Petit, 2003); F_6 to F_{13} are the mean longitudes of the planets (Mercury to Neptune), including the Earth; F_{14} is the general precession in longitude. They are given in radians and T in Julian Centuries of TDB (see Sect. 2.8). The coefficients N_j are functions of index i and can be found in IERS website.

$$\begin{aligned} F_6 &= l_{Me} = 4.402608842 + 2608.7903141574T, \\ F_7 &= l_{Ve} = 3.176146697 + 1021.3285546211T, \\ F_8 &= l_E = 1.753470314 + 628.3075849991T, \end{aligned}$$

$$
\begin{aligned}
F_9 &= l_{Ma} = 6.203480913 + 334.0612426700T, \\
F_{10} &= l_{Ju} = 0.599546497 + 52.9690962641T, \\
F_{11} &= l_{Sa} = 0.874016757 + 21.3299104960T, \\
F_{12} &= l_{Ur} = 5.481293872 + 7.4781598567T, \\
F_{13} &= l_{Ne} = 5.311886287 + 3.8133035638T, \\
F_{14} &= P_a = 0.024381750T + 0.00000538691T^2.
\end{aligned} \tag{2.35}
$$

Using the new paradigm, the complete procedure of transforming the GCRS to the ITRS, which is compatible with the IAU2000 precession-nutation, is based on the expressions of (2.31), (2.32) and (2.33).

An equivalent way to realise the transformation between TRS and CRS under the definition of IAU 2000 can be implemented in a classical way by adding IAU2000 corrections to the corresponding rotating angles. Using the transformation formula (2.14), where the three precession rotating angles (see McCarthy and Petit, 2003) are

$$
\begin{aligned}
z &= -2.5976176'' + 2306.0803226''T + 1.0947790''T^2 \\
&\quad + 0.0182273''T^3 + 0.0000470''T^4 - 0.0000003''T^5, \\
\theta &= 2004.1917476''T - 0.4269353''T^2 - 0.0418251''T^3 \\
&\quad - 0.0000601''T^4 - 0.0000001''T^5
\end{aligned} \tag{2.36}
$$

and

$$
\begin{aligned}
\zeta &= 2.5976176'' + 2306.0809506''T + 0.3019015''T^2 \\
&\quad + 0.0179663''T^3 - 0.0000327''T^4 - 0.0000002''T^5.
\end{aligned}
$$

The IAU 2000 nutation model is given by series for nutation in longitude $\Delta\psi$ and obliquity $\Delta\varepsilon$, referred to the mean equator and equinox of date, with T measured in Julian centuries from epoch J2000.0:

$$
\begin{aligned}
\Delta\psi &= \sum_{i=1}^{N} (A_i + A'_i T)\cos\beta + (A''_i + A'''_i T)\cos\beta, \\
\Delta\varepsilon &= \sum_{i=1}^{N} (B_i + B'_i T)\cos\beta + (B''_i + B'''_i T)\cos\beta,
\end{aligned} \tag{2.37}
$$

where argument β can be found on the IERS website. For these two formulas, rate and bias corrections are necessary because of the new definition of the Celestial Intermediate Pole and the Celestial and Terrestrial ephemeris Origin:

$$
\begin{aligned}
d\Delta\psi &= (-0.0166170 \pm 0.0000100)'' + (-0.29965 \pm 0.00040)''T, \\
d\Delta\varepsilon &= (-0.0068192 \pm 0.0000100)'' + (-0.02524 \pm 0.00010)''T.
\end{aligned} \tag{2.38}
$$

The Earth rotation angle (i.e. the apparent Greenwich Sidereal Time GST or GAST) can be computed by adding a correction EO to the GMST in (2.27) (in mas)

$$\begin{aligned} EO = {} & 14506 + 4612157399.66T + 1396677.21T^2 - 93.44T^3 + 18.82T^4 \\ & + \Delta\psi \cos\varepsilon + \sum_i [(d_{s,0})_i \sin\beta + (d_{c,0})_i \cos\beta] \\ & + \sum_i [(d_{s,1})_i T \sin\beta + (d_{c,1})_i T \cos\beta] + \cdots, \end{aligned} \tag{2.39}$$

where coefficients $(d_{s,j})_i, (d_{c,j})_i$ can be extracted from table5.4 (available at ftp://tai.bipm.org/iers/conv2003/chapter5/). $\Delta\psi$ is defined in (2.37) and ε is defined in (2.18).

Similarly, the rotation matrix of polar motion shall be represented as the first formula of (2.25) and (2.26).

2.6 Geocentric Ecliptic Inertial Coordinate System

As discussed above, ECI uses the CIO pole in the space as the z-axis (through consideration of the polar motion, nutation and precession). If the ecliptic pole is used as the z-axis, then an ecliptic coordinate system is defined, and it may be called the Earth Centred Ecliptic Inertial (ECEI) coordinate system. ECEI places the origin at the mass centre of the Earth, its z-axis is directed to the ecliptic pole (or, the xy-plane is the mean ecliptic), and its x-axis is pointed to the vernal equinox of date. The coordinate transformation between the ECI and ECEI systems can be represented as

$$X_{\text{ECEI}} = R_1(-\varepsilon) X_{\text{ECI}}, \tag{2.40}$$

where ε is the ecliptic angle (mean obliquity) of the ecliptic plane related to the equatorial plane. The formula for ε is given in Sect. 2.4. Usually, coordinates of the sun and the moon, as well as planets, are given in the ECEI system.

2.7 Satellite Fixed Coordinate System

The orbit data, which describes the position of the satellite, is usually referred to the mass centre of the satellite. However, the orbit determination is usually measured through an instrument which is not exactly at the mass centre of the satellite. Therefore, a satellite fixed coordinate system is necessary to be defined for describing the position of the instrument (e.g., antenna or reflector). Such antenna centre correction (also called mass centre correction) has to be applied to the satellite coordinates in precise applications.

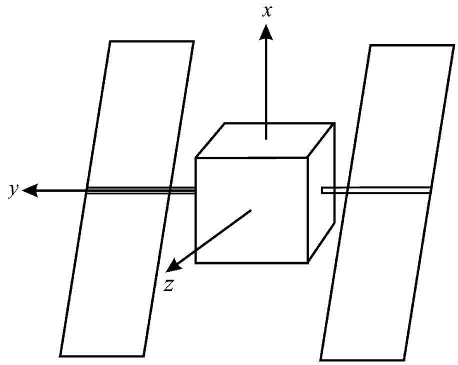


Fig. 2.8 Satellite fixed coordinate system

A satellite fixed coordinate system shall be set up for describing the antenna phase centre offset to the mass centre of the satellite. As shown in Fig. 2.8, the origin of the frame coincides with the mass centre of the satellite, the z-axis is parallel to the antenna pointing direction, the y-axis is parallel to the solar-panel axis, and the x-axis is selected to complete the right-handed frame. A solar vector is a vector from the satellite mass centre pointed to the sun. During the motion of the satellite, the z-axis is always pointing to the Earth, and the y-axis (solar-panel axis) shall be kept perpendicular to the solar vector. In other words, the y-axis is always perpendicular to the plane, which is formed by the sun, the Earth and satellite. The solar-panel can be rotated around its axis to keep the solar-panel perpendicular to the ray of the sun for optimally collecting the solar energy. The solar angle β is defined as the angle between the z-axis and the solar identity vector $\vec{n}_{\text{sun}}$ (see Fig. 2.9). Denoting the identity vector of the satellite fixed frame as ($\vec{e}_x$, $\vec{e}_y$, $\vec{e}_z$), then the solar identity vector can be represented as

$$\vec{n}_{\text{sun}} = \left(\sin\beta,\ 0,\ \cos\beta \right). \tag{2.41}$$

β is needed for computation of the solar radiation pressure in orbit determination.

Denoting $\vec{r}$ as the geocentric satellite vector and $\vec{r}_{\text{s}}$ as the geocentric solar vector (Fig. 2.10),

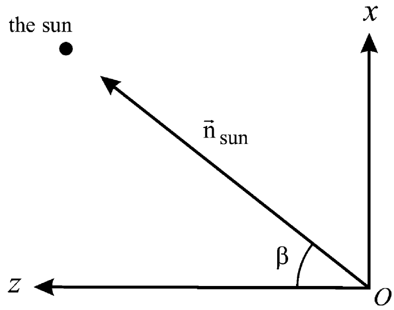


Fig. 2.9 The sun vector in satellite fixed frame

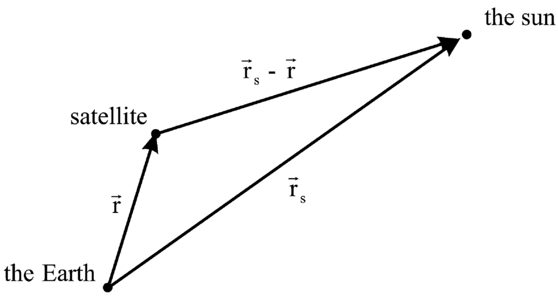


Fig. 2.10 The Earth-sun-satellite vectors

$$\vec{r} = \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}, \qquad \vec{r}_S = \begin{pmatrix} X_{\text{sun}} \\ Y_{\text{sun}} \\ Z_{\text{sun}} \end{pmatrix}, \tag{2.42}$$

then in a geocentric coordinate system one has

$$\vec{e}_z = -\frac{\vec{r}}{|\vec{r}|}, \tag{2.43}$$

$$\vec{e}_y = \frac{\vec{e}_z \times \vec{n}_{\text{sun}}}{|\vec{e}_z \times \vec{n}_{\text{sun}}|},$$

$$\vec{e}_x = \vec{e}_y \times \vec{e}_z, \tag{2.44}$$

$$\vec{n}_{\text{sun}} = \frac{\vec{r}_{\text{s}} - \vec{r}}{|\vec{r}_{\text{s}} - \vec{r}|} \tag{2.45}$$

and

$$\cos\beta = \vec{n}_{\text{sun}} \cdot \vec{e}_z, \tag{2.46}$$

or

$$\vec{e}_z = \frac{-1}{r} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}, \quad r = \sqrt{X^2 + Y^2 + Z^2}, \tag{2.47}$$

$$\vec{n}_{\text{sun}} = \frac{1}{R} \begin{pmatrix} X_{\text{sun}} - X \\ Y_{\text{sun}} - Y \\ Z_{\text{sun}} - Z \end{pmatrix}, \tag{2.48}$$

$$\vec{e}_y = \frac{-1}{S} \begin{pmatrix} YZ_{\text{sun}} - Y_{\text{sun}}Z \\ ZX_{\text{sun}} - Z_{\text{sun}}X \\ XY_{\text{sun}} - X_{\text{sun}}Y \end{pmatrix} \tag{2.49}$$

Table 2.1 GPS satellite antenna phase centre offset

Satellite	x	y	z
Block I	0.2100	0.0	0.8540
Block II/IIA	0.2794	0.0	1.0259
Block IIR	0.0000	0.0	1.2053

and

$$\vec{e}_x = \frac{1}{S \cdot r} \begin{pmatrix} (ZX_{\text{sun}} - Z_{\text{sun}}X)Z - (XY_{\text{sun}} - X_{\text{sun}}Y)Y \\ (XY_{\text{sun}} - X_{\text{sun}}Y)X - (YZ_{\text{sun}} - Y_{\text{sun}}Z)Z \\ (YZ_{\text{sun}} - Y_{\text{sun}}Z)Y - (ZX_{\text{sun}} - Z_{\text{sun}}X)X \end{pmatrix}, \tag{2.50}$$

where

$$R = \sqrt{(X_{\text{sun}} - X)^2 + (Y_{\text{sun}} - Y)^2 + (Z_{\text{sun}} - Z)^2} \tag{2.51}$$

and

$$S = \sqrt{(YZ_{\text{sun}} - Y_{\text{sun}}Z)^2 + (ZX_{\text{sun}} - Z_{\text{sun}}X)^2 + (XY_{\text{sun}} - X_{\text{sun}}Y)^2}. \tag{2.52}$$

Suppose the satellite antenna phase centre in the satellite fixed frame is (x,y,z), then the offset vector in the geocentric frame can be obtained by substituting (2.47), (2.49) and (2.50) into the following formula:

$$\vec{d} = x\vec{e}_x + y\vec{e}_y + z\vec{e}_z, \tag{2.53}$$

which may be added to the vector $\vec{r}$.

GPS satellite antenna phase centre offsets in the satellite fixed frame are given in Table 2.1.

The dependence of the phase centre on the signal direction and frequencies is not considered for the satellite here. A mis-orientation of the $\vec{e}_y$ ($\vec{e}_x$ too) of the satellite with respect to the sun may cause errors in the geometrical phase centre correction. In the Earth's shadow (for up to 55 min), the mis-orientation becomes worse. The geometrical mis-orientation may be modelled and estimated.

2.8 Time Systems

The three time systems used in satellite surveying are sidereal time, dynamic time and atomic time (see, e.g., Hofmann-Wellenhof et al., 1997/2001; Leick, 1995/2004; McCarthy, 1996; King et al., 1987).

Sidereal time is a measure of the Earth's rotation and is defined as the hour angle of the vernal equinox. If the measure is counted from the Greenwich meridian, the sidereal time is called Greenwich Sidereal Time. Universal Time (UT) is the Greenwich hour angle of the apparent sun, which is orbiting uniformly in the equatorial plane. Because the angular velocity of the Earth's rotation is not a constant, sidereal time is not a uniformly-scaled time. The oscillation of UT is also partly caused by the polar motion of the Earth. The universal time corrected for the polar motion is denoted by UT1.

Dynamical time is a uniformly-scaled time used to describe the motion of bodies in a gravitational field. Barycentric Dynamic Time (TDB) is applied in an inertial coordinate system (its origin is located at the centre-of-mass (Barycentre)). Terrestrial Dynamic Time (TDT) is used in a quasi-inertial coordinate system (such as ECI). Because of the motion of the Earth around the sun (or say, in the sun's gravitational field), TDT will have a variation with respect to TDB. However, both the satellite and the Earth are subject to almost the same gravitational perturbations. TDT may be used for describing the satellite motion without taking into account the influence of the gravitational field of the sun. TDT is also called Terrestrial Time (TT).

Atomic Time is a time system kept by atomic clocks such as International Atomic Time (TAI). It is a uniformly-scaled time used in the ECEF coordinate system. TDT is realised by TAI in practice with a constant offset (32.184 s). Because of the slowing down of the Earth's rotation with respect to the sun, Coordinated Universal Time (UTC) is introduced to keep the synchronisation of TAI to the solar day (by inserting the leap seconds). GPS Time (GPST) is also atomic time.

The relationships between different time systems are given as follows:

$$\begin{aligned} \mathrm{TAI} &= \mathrm{GPST} + 19.0\,\mathrm{sec}, \\ \mathrm{TAI} &= \mathrm{TDT} - 32.184\,\mathrm{sec}, \\ \mathrm{TAI} &= \mathrm{UTC} + n\,\mathrm{sec} \\ \mathrm{UT1} &= \mathrm{UTC} + \mathrm{dUT1}, \end{aligned} \tag{2.54}$$

where dUT1 can be obtained by IERS, ($\mathrm{dUT1} < 0.7\,\mathrm{s}$, see Zhu et al., 1996), (dUT1 is also broadcasted with the navigation data), n is the number of leap seconds of date and is inserted into UTC on the 1st of January and 1st of July of the years. The actual n can be found in the IERS report.

Time argument T (Julian centuries) is used in the formulas given in Sect. 2.4. For convenience, T is denoted by TJD, and TJD can be computed from the civil date (Year, Month, Day, and Hour) as follows:

$$\mathrm{JD} = \mathrm{INT}(365.25Y) + \mathrm{INT}(30.6001\,(M+1)) + \mathrm{Day} + \mathrm{Hour}/24 + 1720981.5$$

and

$$\mathrm{TJD} = \mathrm{JD}/36525, \tag{2.55}$$

where

$$\begin{aligned} &Y = \mathrm{Year} - 1, \quad M = \mathrm{Month} + 12, \quad \text{if Month} \le 2, \\ &Y = \mathrm{Year}, \qquad\;\; M = \mathrm{Month}, \qquad\quad\; \text{if Month} > 2, \end{aligned}$$

where JD is the Julian Date, Hour is the time of UT and INT denotes the integer part of a real number. The Julian Date counted from JD2000.0 is then JD2000 = JD–JD2000.0, where JD2000.0 is the Julian Date of 2000 January 1st 12h and has the value of 2451 545.0 days. One Julian century is 36 525 days.

Inversely, the civil date (Year, Month, Day and Hour) can be computed from the Julian Date (JD) as follows:

$$\begin{aligned}
b &= \mathrm{INT}(\mathrm{JD}+0.5)+1537, \\
c &= \mathrm{INT}\left(\frac{b-122.1}{365.25}\right), \\
d &= \mathrm{INT}(365.25c), \\
e &= \mathrm{INT}\left(\frac{b-d}{30.6001}\right), \\
\mathrm{Hour} &= \mathrm{JD}+0.5-\mathrm{INT}(\mathrm{JD}+0.5), \\
\mathrm{Day} &= b-d-\mathrm{INT}(30.6001e), \\
\mathrm{Month} &= e-1-12\mathrm{INT}\left(\frac{e}{14}\right)
\end{aligned}$$

and

$$\mathrm{Year} = c-4715-\mathrm{INT}\left(\frac{7+\mathrm{Month}}{10}\right), \tag{2.56}$$

where b, c, d, and e are auxiliary numbers.

Because the GPS standard epoch is defined as JD = 2444244.5 (1980 January 6, 0h), GPS week and the day of week (denoted by Week and N) can be computed by

$$N = \mathrm{modulo}(\mathrm{INT}(\mathrm{JD}+1.5),7)$$

and

$$\mathrm{Week} = \mathrm{INT}\left(\frac{\mathrm{JD}-2444244.5}{7}\right), \tag{2.57}$$

where N is the day of week ($N = 0$ for Monday, $N = 1$ for Tuesday, and so on).

For saving digits and counting the date from midnight instead of noon, the Modified Julian Date (MJD) is defined as

$$\mathrm{MJD} = (\mathrm{JD}-2400000.5). \tag{2.58}$$

GLONASS time (GLOT) is defined by Moscow time $\mathrm{UTC_{SU}}$, which equals UTC plus three hours (corresponding to the offset of Moscow time to Greenwich time), theoretically. GLOT is permanently monitored and adjusted by the GLONASS Central Synchroniser (see Roßbach, 2006). UTC and GLOT then have a simple relation

$$\mathrm{UTC} = \mathrm{GLOT}+\tau_\mathrm{c}-3\mathrm{h},$$

where τ_c is the system time correction with respect to UTC_{SU}, which is broadcasted by the GLONASS ephemeris and is less than one microsecond. Therefore there is approximately

$$\mathrm{GPST} = \mathrm{GLOT} + m - 3\mathrm{h},$$

where m is the number of “leap seconds” between GPS and GLONASS (UTC) time and is given in the GLONASS ephemeris. m is indeed the leap seconds since GPS standard epoch (1980 January 6, 0h).

Galileo system time (GST) will be maintained by a number of UTC laboratory clocks. GST and GPST are time systems of various UTC laboratories. After the offset of GST and GPST is made available to the user, the interoperability will be ensured.

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