**Details on the subroutine ropp\_pp\_geometric\_optics**

An initial approximation of U\_gns and U\_leo is given as:

U\_leo(:) = (r\_leo(:) - r\_gns(:))/SQRT(SUM((r\_leo(:) - r\_gns(:))\*\*2))

U\_gns(:) = U\_leo(:)

Few iterations follows to obtain U\_leo and U\_gns. It means that 6 unknowns have to be computed. It is based on the following observation equation:

Observation equation based on Doppler:

DF(1) = doppler - (DOT\_PRODUCT(v\_gns,U\_gns)-DOT\_PRODUCT(v\_leo,U\_leo)) /(c\_light - DOT\_PRODUCT(v\_gns,U\_gns))

There are few other (5) conditions to be attended: (DF(:) is expected to be zero.

DF(2:4) = vector\_product(r\_gns, U\_gns) - vector\_product(r\_leo, U\_leo)

DF(5) = 1.0\_wp - DOT\_PRODUCT(U\_leo,U\_leo)

DF(6) = 1.0\_wp - DOT\_PRODUCT(U\_gns,U\_gns)

Therefore, we have 6 equations and 6 unknowns to be solved.

As we have a non-linear system, we have to linearized and apply iterations around the approximate values U\_leo and U\_gns. Accordingly with ROPP algorithms we have:

A(1,1:3) = -v\_leo/(c\_light - DOT\_PRODUCT(v\_leo,U\_leo))

A(1,4:6) = v\_gns \* (c\_light - DOT\_PRODUCT(v\_leo,U\_leo)) / &

(C\_Light - DOT\_PRODUCT(v\_gns,U\_gns))\*\*2

DO i=1,3

DO j=1,3

A(i+1,j) = SUM(tensor(i,:,j)\*r\_leo(:))

A(i+1,j+3) = -SUM(tensor(i,:,j)\*r\_gns(:))

END DO

END DO

A(5,1:3) = 2.0\_wp\*U\_leo

A(6,4:6) = 2.0\_wp\*U\_gns

Apparently there is a problem with this Jacobian A matrix. From my deduction, and further discussion with ROPP tem, I obtained the following:

A(1,1:3) = -v\_leo/(c\_light - DOT\_PRODUCT(v\_gns,U\_gns)).

For the others, no detected changes anymore.

A(1,4:6) = v\_gns \* (c\_light - DOT\_PRODUCT(v\_leo,U\_leo)) / &

(C\_Light - DOT\_PRODUCT(v\_gns,U\_gns))\*\*2

DO i=1,3

DO j=1,3

A(i+1,j) = SUM(tensor(i,:,j)\*r\_leo(:))

A(i+1,j+3) = -SUM(tensor(i,:,j)\*r\_gns(:))

END DO

END DO

A(5,1:3) = 2.0\_wp\*U\_leo

A(6,4:6) = 2.0\_wp\*U\_gns

**RESULTS INFLUENCE**

Few tests will be carried out to test any influence on the final results, mainly on impact parameters and refractivity (N).